SEMICONDUCTOR JUNCTION CIRCULATORS

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ABSTRACT

Modern ferrite circulators operating above 40GHz, are narrowband devices, due to the ferrite materials available today. Tracking operation of junction circulators above 60GHz are, in principle possible employing magnetoplasmons.

Theoretical results are presented showing broadband operation for idealised Gallium Arsenide cooled to 77K, and an example is shown for the frequency range 92-100 GHz. A narrowband, solution is also given where the losses due to electron collisions are modelled. The circulation conditions in the latter example are designed using intersecting impedance curves rather than the tracking solutions achieved in the lossless case.

I INTRODUCTION

Analysis of the junction circulator employing ferrites has been well understood [1, 2]. Such ferrite designs provide broad-band, impedance tracking performance up to 40GHz. Above this frequency ferrite designs are limited to narrowband operation, since the maximum saturation magnetisation available is about 5500 Gauss. In this paper broadband junction circulators [3] are predicted by applying the Green's function approach to the Drude-Zener model of semiconductors.

The semiconductor is initially assumed to be lossless, that is the collision frequency is zero. Here, the semiconductor used is idealised gallium arsenide cooled to 77K. The scattering parameters are derived for the magnetoplasma and from these the bandwidth and circulation equations are derived for the electrical equivalent of the magnetised ferrite. The semiconductor gives rise to a permittivity tensor instead of the permeability tensor experienced in ferrites.

II HELMHOLTZ WAVE EQUATION

A semiconductor magnetised in the z-direction possesses the tensor permittivity in cylindrical coordinates as given by equation 1.

$$[\epsilon] = \begin{bmatrix} \varepsilon & -j\kappa & 0 \\ +j\kappa & \varepsilon & 0 \\ 0 & 0 & \zeta \end{bmatrix} \tag{1}$$

Applying this to Maxwell's equations and assuming a temporal variation of $e^{j\omega t}$ gives:

$$\nabla \times \mathbf{E} = -j\omega\mu_o \mathbf{H} \tag{2}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon_o [\epsilon] \mathbf{E} \tag{3}$$

Here μ_o is the permeability of the semiconductor.

It is supposed that the semiconductor disk is sufficiently thin so that there is no field variation in the disk in the z-direction. The problem is then reduced to two of three dimensions. From expanding Maxwell's equations two sets of solutions emerge:

TM modes consisting of E_z, H_r and H_ϕ. These have no magnetic dependence (and hence no κ).

TE modes comprising H_z, E_r and E_ϕ. These are of interest here and correspond to the TM modes found in the magnetised ferrite case.

Further manipulation of the TE mode solutions gives the Helmholtz equation in H_z of

$$\frac{\partial H_z{}^2}{\partial^2 r} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \frac{1}{r^2}\frac{\partial H_z{}^2}{\partial^2 \phi} + k^2 H_z = 0 \tag{4}$$

where,

$$k^2 = \omega^2 \mu_o \varepsilon_o \varepsilon_{eff} \tag{5}$$

and the effective permittivity is

$$\varepsilon_{eff} = \frac{\varepsilon^2 - \kappa^2}{\varepsilon} \tag{6}$$

The general solution to this equation for $\varepsilon_{eff} \geq 0$ and finite magnetic field at zero radius is given by:

$$H_{z,n}(r,\phi) = a_n J_n(kr) e^{jn\phi} \tag{7}$$

where,

a_n are constants,
J_n is the nth order Bessel function of the first kind.

Expressions for E_ϕ and E_r can be written as,

$$E_\phi = -\frac{1}{j\omega\varepsilon_o\varepsilon_{eff}}\left\{j\frac{\kappa}{\varepsilon}.\frac{1}{r}\frac{\partial H_z}{\partial\phi} + \frac{\partial H_z}{\partial r}\right\} \quad (8)$$

$$E_r = \frac{1}{j\omega\varepsilon_o\varepsilon_{eff}}\left\{\frac{1}{r}.\frac{\partial H_z}{\partial\phi} - j\frac{\kappa}{\varepsilon}\frac{\partial H_z}{\partial r}\right\} \quad (9)$$

III BOUNDARY CONDITIONS

The boundary conditions are such that at a radius R an electric wall is present except at the three ports illustrated in figure 1. At each port a transmission line is connected in such a way that an electric field E_ϕ and a magnetic field H_z are present. These fields are assumed constant over the angle subtended at each port 2ψ. On top and bottom of the disk there are magnetic walls.

The boundary conditions are summarised in figure 1 and equation 10.

$$E_\phi(R,\phi) = \begin{cases} A & -\frac{\pi}{3}-\psi < \phi < -\frac{\pi}{3}+\psi \\ B & \frac{\pi}{3}-\psi < \phi < \frac{\pi}{3}+\psi \\ C & \pi-\psi < \phi < \pi+\psi \\ 0 & elsewhere \end{cases} \quad (10)$$

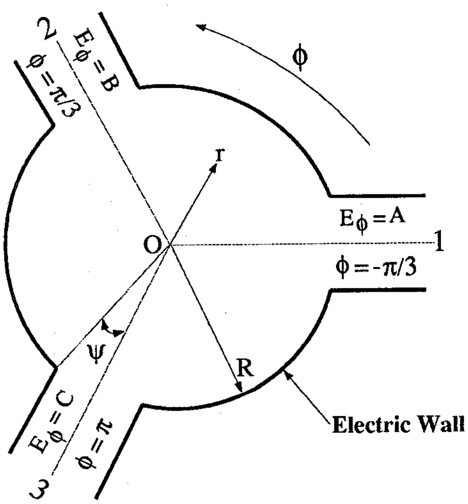

Figure 1: Boundary conditions on the semiconductor disk

Now if $E_\phi(r,\phi)$ is known at the edge by virtue of equation 10 then H_z is determined by oblique boundary conditions. A Green's function $G(r,\phi;r',\phi')$ can be introduced such that:

$$H_z(r,\phi) = \int_{-\pi}^{\pi} G(r,\phi;r',\phi')E_\phi(R,\phi')d\phi' \quad (11)$$

and is defined

$$G(\phi,\phi') \equiv G(R,\phi,R,\phi') \quad (12)$$

or for the boundary conditions:

$$\begin{aligned} H_z(r,\phi) &= \int_{-\frac{\pi}{3}-\psi}^{-\frac{\pi}{3}+\psi} G(\phi;\phi')\,Ad\phi' + \int_{\frac{\pi}{3}-\psi}^{\frac{\pi}{3}+\psi} G(\phi;\phi')\,Bd\phi' \\ &+ \int_{\pi-\psi}^{\pi+\psi} G(\phi;\phi')\,Cd\phi' \end{aligned}$$

and the averaged magnetic field across the ports is given by:

$$\begin{aligned} port1: H_{z_1} &= \frac{1}{2\psi}\int_{-\frac{\pi}{3}-\psi}^{-\frac{\pi}{3}+\psi} H_z(\phi)d\phi \\ port2: H_{z_2} &= \frac{1}{2\psi}\int_{\frac{\pi}{3}-\psi}^{\frac{\pi}{3}+\psi} H_z(\phi)d\phi \\ port3: H_{z_3} &= \frac{1}{2\psi}\int_{\pi-\psi}^{\pi+\psi} H_z(\phi)d\phi \end{aligned} \quad (13)$$

since, A, B and C are constants over the aperture width, only the Green's function needs to be integrated.

The integrated Green's function will be defined:

$$\bar{G}(\phi;\phi') = \frac{1}{2\psi}\int_{\phi-\psi}^{\phi+\psi}\int_{\phi'-\psi}^{\phi'+\psi} G(\phi;\phi')\,d\phi' d\phi \quad (14)$$

IV THE GREEN'S FUNCTION

Again following Bosma's analysis from equations 7, 8 and 9 E_ϕ and E_r can be determined. Here the Green's function is:

$$\begin{aligned} G\quad(\phi,\phi') &= -\frac{jJ_o(x)}{2\pi Z_{eff}J_o'(x)} \\ &+ \frac{1}{\pi Z_{eff}}\sum_{n=1}^{\infty}\frac{\frac{\kappa}{\varepsilon}.\frac{n}{x}J_n^2(x)\,sin[n\theta] - jJ_n'(x)\,J_n(x)\,cos[n\theta]}{\{J_n'(x)\}^2 - \{\frac{\kappa}{\varepsilon}\frac{n}{x}J_n(x)\}^2} \end{aligned} \quad (15)$$

where, $x = kR$ (R is the disk radius), $\theta = \phi - \phi'$ and

$$Z_{eff} = \sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_{eff}}} \quad (16)$$

The integral of this function is given by substituting equation 15 into 14.

V SCATTERING PARAMETERS

Defining,

$$\begin{aligned} \bar{G}_1 &= \bar{G}\left(-\frac{\pi}{3};-\frac{\pi}{3}\right) \\ \bar{G}_2 &= \bar{G}\left(-\frac{\pi}{3};\frac{\pi}{3}\right) \\ \bar{G}_3 &= \bar{G}\left(-\frac{\pi}{3};\pi\right) \end{aligned} \quad (17)$$

Then the magnetic field present at each port is given by:

$$\begin{aligned} H_{z_1} &= A\bar{G}_1 + B\bar{G}_2 + C\bar{G}_3 \\ H_{z_2} &= A\bar{G}_3 + B\bar{G}_1 + C\bar{G}_2 \\ H_{z_3} &= A\bar{G}_2 + B\bar{G}_3 + C\bar{G}_1 \end{aligned} \quad (18)$$

Considering the input port to be port 1 and ports 2 and 3 are terminated in reflectionless loads. Then the parts of a wave incident on the input port which are dissipated in the terminations are a measure of the transmission and reflection coefficients. Now, only outward waves are present at the ports 2 and 3, hence,

$$\frac{H_{z_2}}{B} = Y_d \tag{19}$$

and,

$$\frac{H_{z_3}}{C} = Y_d \tag{20}$$

where, Y_d is the stripline or finline admittance. This differs from Bosma's solution due to the orientation of the E and H fields. From these equations and substituting into equations 18, H_{z_1} can be expressed purely in terms of $\bar{G}_1$, $\bar{G}_2$ and $\bar{G}_3$ alone.

The scattering matrix of a lossless, cyclic symmetrical 3-port may be written as:

$$[S] = \begin{bmatrix} \alpha & \gamma & \beta \\ \beta & \alpha & \gamma \\ \gamma & \beta & \alpha \end{bmatrix} \tag{21}$$

and the reflection coefficient α, the isolation coefficient β and through parameter γ can now be derived. Making the following substitutions :

$$\begin{aligned} \bar{G}_1 - Y_d &= \frac{2Q_1}{j\pi Z_{eff}} \\ \bar{G}_2 &= \frac{2Q_2}{j\pi Z_{eff}} \\ \bar{G}_3 &= \frac{2Q_3}{j\pi Z_{eff}} \end{aligned} \tag{22}$$

and performing the necessary substitutions for $\bar{G}_n$ for the scattering parameters gives:

$$\begin{aligned} \alpha &= -1 - \frac{j\pi Z_{eff} Y_d \left[Q_1^2 - Q_2 Q_3\right]}{\left[Q_1^3 + Q_2^3 + Q_3^3 - 3Q_1Q_2Q_3\right]} \\ \beta &= -\frac{j\pi Z_{eff} Y_d \left[Q_2^2 - Q_1 Q_3\right]}{\left[Q_1^3 + Q_2^3 + Q_3^3 - 3Q_1Q_2Q_3\right]} \\ \gamma &= -\frac{j\pi Z_{eff} Y_d \left[Q_3^2 - Q_1 Q_2\right]}{\left[Q_1^3 + Q_2^3 + Q_3^3 - 3Q_1Q_2Q_3\right]} \end{aligned} \tag{23}$$

For perfect circulation one of the scattering matrix elements (β or γ) must be zero, here β is chosen. This gives :

$$Q_2^2 = Q_1 Q_3 \tag{24}$$

Now if Q_1, Q_2 and Q_3 are separated into real and imaginary parts thus,

$$\begin{aligned} Q_1 &= R + jS \\ Q_2 &= V + jW \\ Q_3 &= V - jW \end{aligned} \tag{25}$$

Then substituting into equation 24 the circulation equations are,

$$S = W\frac{(3V^2 - W^2)}{V^2 + W^2} \tag{26}$$

and,

$$R = V\frac{(V^2 - 3W^2)}{V^2 + W^2} \tag{27}$$

where,

$$\begin{aligned} R &= \frac{\psi J_o(x)}{2J_o'(x)} + \sum_{n=1}^{\infty} \frac{sin^2(n\psi)}{n^2\psi} \frac{J_n(x)\,J_n'(x)}{\{J_n'(x)\}^2 - \{\frac{\kappa}{\varepsilon}\frac{n}{x}J_n(x)\}^2} \\ S &= -\frac{\pi Z_{eff}}{2Z_d} \\ V &= \frac{\psi J_o(x)}{2J_o'(x)} + \sum_{n=1}^{\infty} \frac{sin^2(n\psi)}{n^2\psi} \frac{J_n'(x)\,J_n(x)\,cos\left(\frac{2\pi n}{3}\right)}{\{J_n'(x)\}^2 - \{\frac{\kappa}{\varepsilon}\frac{n}{x}J_n(x)\}^2} \\ W &= -\sum_{n=1}^{\infty} \frac{sin^2(n\psi)}{n^2\psi} \frac{\frac{\kappa n}{\varepsilon x}J_n^2(x)\,sin\left(\frac{2\pi n}{3}\right)}{\{J_n'(x)\}^2 - \{\frac{\kappa}{\varepsilon}\frac{n}{x}J_n(x)\}^2} \end{aligned} \tag{28}$$

R, V and W are an infinite series with each term corresponding to a particular mode.

With reference to Wu and Rosenbaum's results for a ferrite circulator the circulation equations are very similar to those derived here, except for a sign change in S and W and μ is replaced by ε. The major difference is the second circulation equation will give a curve of Z_d/Z_{eff} versus κ/ε as plotted in figure 2 instead of Z_{eff}/Z_d versus κ/ε as produced in the ferrite case.

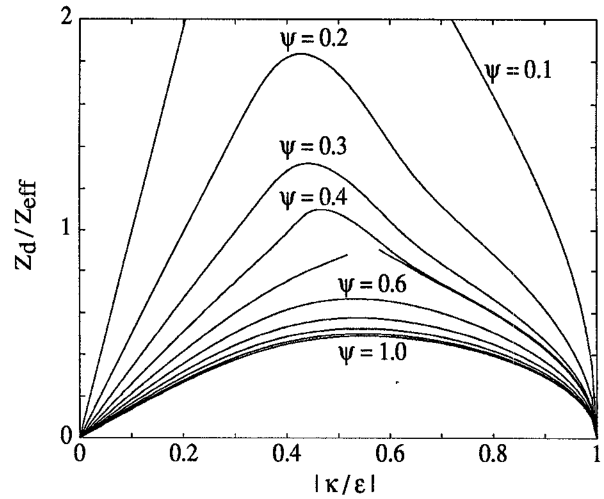

Figure 2: Solution of Second Circulation Condition, for various values of coupling angle

VI BROADBAND DESIGN EXAMPLE

The frequency of operation of this device will be above the extraordinary wave frequency, for the medium, f_{res}, where,

$$f_{res} = \left(\frac{f_p^2}{\sqrt{\varepsilon_r}} + f_c^2\right)^{\frac{1}{2}} \tag{29}$$

here,

f_p is the plasma frequency, (Hz) and
f_c is the cyclotron frequency, (Hz) and
ε_r is the static dielectric constant

The semiconductor of interest is gallium arsenide cooled to 77K which has a mid range value for ε_r of 12.0. If the d.c. applied magnetic field $B_o = 0.12$T and the electron concentration is $3.6 \times 10^{13} cm^{-3}$ then $f_{res} = 78.1$GHz. The impedance ratio Z_d/Z_{eff} is given by the roots of the circulation equation 26 and by the definition:

$$\frac{Z_d}{Z_{eff}} = \left[\frac{\varepsilon}{\varepsilon_d}\right]^{\frac{1}{2}} \left[1 - \left(\frac{\kappa}{\varepsilon}\right)^2\right]^{\frac{1}{2}} \qquad (30)$$

where ε_d is the dielectric constant of the surrounding medium.

Choosing ε_d=10.0. Then by careful choice of the half angle ψ, the two curves for Z_d/Z_{eff} may be matched to facilitate a continuous solution indicating a broadband circulator.

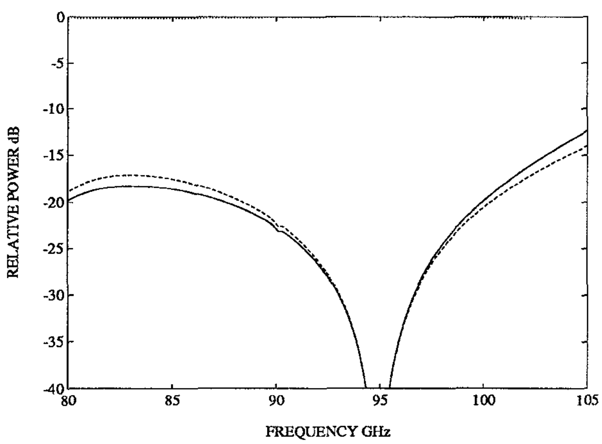

Figure 3: Predicted Performance of Idealised GaAs Circulator at 77K [Return Loss ———— , Isolation - - - - - - - - - - - - , Insertion Loss ············]

Neglecting losses due to electron collisions one such design gave ψ=0.7 radians and a radius of 0.361mm.This gives good theoretical circulation performance from 90GHz to 99GHz. Calculating the S-parameters using this data gives the response illustrated in figure 3. This indicates a 25dB isolation bandwidth of approximately 91.5-98GHz, i.e. 6.9%. This is larger than that obtainable with a ferrite with $4\pi M_s = 5000$G. It is not as broad as the relative bandwidths available with ferrites operating in the "tracking range" at lower frequencies. This would be discussed in a fuller paper.

VII NARROWBAND DESIGN EXAMPLE INCLUDING LOSSES

The analysis given above is being extended to include losses due to electron collisions, via the electron collision frequency, $\nu_c\ s^{-1}$. This design example is chosen to operate at 40GHz. This is below the extraordinary wave frequency of the medium and has narrowband frequency performance since this design fulfills the circulation conditions only at the intercept of the impedance curves. A preliminary result is shown in figure 4. The insertion loss at 40GHz is -0.82dB.

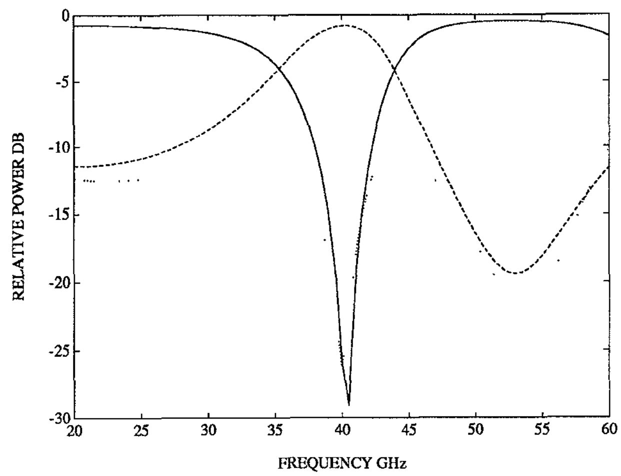

Figure 4: Predicted Performance of GaAs Circulator at 77K including a collision frequency of $\nu_c = 1.3 \times 10^{11} s^{-1}$
[Return Loss ———— ,Isolation ············ ,
Insertion Loss - - - - - - - - - - - -]

This design has a plasma frequency of 100GHz; an applied d.c., magnetic field of 0.4T; a radius of 0.64mm; and a half angle of 0.12 radians.

VIII CONCLUSIONS

A semiconductor junction circulator has been analysed and a predicted 25dB isolation bandwidth of 6% at 94GHz has been shown using idealised, lossless GaAs at 77K. A narrowband, lossy, theoretical design example at 40GHz shows that losses do reduce performance but not to such a degree as to render the device useless. This technology may be compatible with high-T_c superconductors and with MMIC's.

References

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[3] L. E. Davis. Semiconductor junction circulators. *Patent Application Number : 9222553.1*, October 1992.